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Vibration Damping by a Nearly Continuous Distribution of Nearly Undamped Oscillators

by

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It has been claimed that vibration damping can be derived from the coupling of a continuous distribution of undamped oscillators. This claim stems from the fact that the contribution to the damping of a master oscillator by a coupled set of continuously distributed satellite oscillators, is independent of the loss factors of the oscillators in this set. (The distribution is with respect to the frequency of resonance of the satellite oscillators in the set.) The transition from a discrete-to-a continuous distribution, however, cannot be achieved without the imposition of modal overlap on the distribution of the satellite oscillators. It is this imposition that ensures that the contribution to the damping by these satellite oscillators is intuitively real. The imposition forbids equating the loss factors of the satellite oscillators to zero just because their contribution, to the surrogate damping of the host master oscillator to which they are coupled, is independent of these loss factors. Notwithstanding that the quantification of this contributed damping in terms of a dimensionless ratio of dynamic quantities does not uniquely qualify it as a loss factor. Moreover, the analysis of nearly continuous distribution of nearly undamped satellite oscillators brings insights into the manner by which they contribute damping to the mechanical system of which they are an integral part. In part, these insights are obscured in an analysis that is based on the a priori introduction of extreme limits. Indeed, these insights may be the overwhelming justification for the present paper.

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Abstract

It has been claimed that vibration damping can be derived from the coupling of a continuous distribution of undamped oscillators. This claim stems from the fact that the contribution to the damping of a master oscillator by a coupled set of continuously distributed satellite oscillators, is independent of the loss factors of the oscillators in this set. (The distribution is with respect to the frequency of resonance of the satellite oscillators in the set.) The transition from a discrete-to-a continuous distribution, however, cannot be achieved without the imposition of modal overlap on the distribution of the satellite oscillators. It is this imposition that ensures that the contribution to the damping by these satellite oscillators is intuitively real. The imposition forbids equating the loss factors of the satellite oscillators to zero just because their contribution, to the surrogate damping of the host master oscillator to which they are coupled, is independent of these loss factors. Notwithstanding that the quantification of this contributed damping in terms of a dimensionless ratio of dynamic quantities does not uniquely qualify it as a loss factor. Moreover, the analysis of nearly continuous distribution of nearly undamped satellite oscillators brings insights into the manner by which they contribute damping to the mechanical system of which they are an integral part. In part, these insights are obscured in an analysis that is based on the a priori introduction of extreme limits. Indeed, these insights may be the overwhelming justification for the present paper.

Introduction

Central to the present paper is a recent publication entitled "Vibration damping by a continuous distribution of undamped oscillators" by R.J. Nagem, I. Veljkovic and S. Sandri [1]. It is argued in this publication that damping is provided to a host master oscillator by a set of oscillators that are a priori devoid of damping [1]. Although these arguments are well taken and are well presented, the idea that damping can be provided by lossless oscillators is contrary to intuition and needs careful consideration. Cavalierly stating that "the set of continuously distributed oscillators considered here may also be interpreted as a 'heat reservoir' ..." is hardly an adequate consideration. Neither is the argument adequate that the idea and similar interpretive schemes are not new; they had been advanced in a number of preceding publications [2-4]. Nonetheless, the present paper is not seeking to challenge the notion that the damping, that is provided to the master oscillator by a distribution of these lossless oscillators, violates somehow the *balance of power*; i.e., *the law of conservation of energy*. (In the present analysis the question: "Where did the energy go?" does not arise; the question is rendered moot by asymptotically evaluated dynamic quantities when parametric limits are extremely reached for.) Rather, a fundamental initial oversight that beset all these publications is brought to the attention of the reader. Admittedly, once that oversight is addressed, the remaining analyses and arguments in all these publications are largely validated; e.g., validated is the argument that a specifically defined measure of the damping that is provided to the host master oscillator by a distribution of nearly lossless oscillators, is independent of the loss factors that account for the dampings in these oscillators. This independence does not, however, imply that individual loss factors that are equal to zero are admissible; indeed, individual loss factors that are not equal to zero are an essential condition on the validity of the analysis in the first place. Moreover, the analysis of nearly continuous distribution of nearly undamped satellite oscillators brings insights into the manner by which the satellite oscillators contribute damping to the mechanical system comprising a hosting master oscillator and these satellite oscillators. Thus, in the asymptotic evaluations of the balance of power, power is preserved. As the loss factors of the individual

satellite oscillators are made to approach zero, the relative stored energy in these oscillators is increased proportionately so that this relative stored energy approaches infinity. The dissipated power in the satellite oscillators, relative to the external input power into the mechanical system, therefore remains unchanged. This, for example, explains why the damping, of the master oscillator due to the presence of the satellite oscillators, is independent of their assigned loss factors. In part, these insights are obscured in an analysis that is subjected a priori to limiting asymptotic conditions of continuity and distribution; notwithstanding that such impositions may require careful consideration if apparent singular behaviors of the mechanical system are to be avoided.

I. Transition for a discrete-to-a continuous distribution

Consider the mechanical system comprising a master oscillator, with mass (M) and stiffness (K), that is coupled to a set of satellite oscillators. This mechanical system is sketched in Fig. 1 [1–4]. The $(n)th$ satellite oscillator is defined by a mass (m_n) and a stiffness (k_n). The damping is assumed to be associated with the stiffness elements

$$K = K_o(1+i\eta_o) \; ; \qquad (K_o / M) = \omega_o^2 \; , \tag{1a}$$

$$k_n = k_{on}(1+i\eta_n) \; ; \qquad (k_{on} / m_n) = \omega_n^2 \; , \tag{1b}$$

where (η_o) and (η_n) are loss factors. Each of these loss factors characterizes an individual oscillator and quantifies the dissipation that the oscillator, in isolation, can handle when externally driven. The linear equations of motion of the master oscillator in situ and a typical satellite oscillator in situ are

$$[(i\omega M)+(K / i\omega)]\, V_o(\omega) \; + \; \sum_1^N (k_n / i\omega)\,[V_o(\omega) - V_n(\omega)] \; = \; P_e(\omega) \; , \tag{2}$$

$$(i\omega m_n) \; V_n(\omega) \; + \; (k_n / i\omega) \; [V_n(\omega) \; - \; V_o(\omega)] \; = \; 0 \; , \tag{3}$$

respectively, where $V_o(\omega)$ and $V_n(\omega)$ are the response of the master oscillator and the $(n)th$ satellite oscillator, respectively, and $P_e(\omega)$ is the drive that is assumed applied externally to the master oscillator; the satellite oscillators are not driven externally [1-4]. [cf. Fig. 1.] The summation in Eq. (2) is over a set of satellite oscillators; (N) is the number of satellite oscillators in the set. In the publications of reference, a transition is performed or implied from the *summation* $(\sum_1^N \cdots)$, accounting for a discrete distribution, to the *integration* $(\int_o^1 d\xi \cdots)$,

accounting for an imposition of a continuous distribution [1,5,6]. To validate this transition, a condition must be imposed. To establish this condition a modal density is defined in the form

$$(k_{on} / m_n) = \omega_n^2 \ ; \qquad \sum_1^N A_n \rightarrow \int_o^1 d\xi A(\xi) \ ; \qquad 0 < \xi < 1 \ , \tag{4a}$$

$$[k_o(\xi) / m(\xi)] = \omega^2(\xi) \ ; \qquad n(\xi) = (N) \ [\partial\omega(\xi) / \partial\xi]^{-1} \ . \tag{4b}$$

The modal density $n(\xi)$ of a set of satellite oscillators is a function of the dimensionless and continuous variable (ξ), which has values in the interval between zero and unity [1]. The transition from the discrete-to-the continuous is necessarily valid if a *modal overlap* prevails [7]. A modal overlap prevails provided

$$n(\xi)\omega\eta(\xi,\omega) \gtrsim 1 \ ; \qquad \eta_n(\omega) \rightarrow \eta(\xi,\omega) \ . \tag{5}$$

Therefore, the transition stated in Eq. (4a) is validated by the condition

$$\eta(\xi,\omega) \gtrsim [\omega n(\xi)]^{-1} \ ; \qquad [N\eta(\xi,\omega)] \gtrsim [\partial\omega(\xi) / (\omega\partial\xi)] \ , \tag{6a}$$

where Eq. (4b) is substituted in Eq. (5). (A recent investigation, under Reference 8, reveals that if the introduction of a set of satellite oscillators is intended to achieve viable noise control benefits, then an optimal modal overlap condition requires that $[n(\omega)\,\omega\,\eta(\xi,\omega)] \simeq (b)$ or equivalently that $[N\eta(\xi,\omega)] \simeq (b)[\partial\omega(\xi)/(\omega\partial\xi)]^{-1}$, with $(b) \simeq (3/2)$ and (N) large; e.g., $N = 10$. An exaggerated inequality in Eq. (6a); e.g., $(b) = (8)$, may lead to an erosion in the noise control viability of the set. However, in the investigation under Reference 8, (N) is *numerable*; i.e., $N \lesssim 20$, whereas in the present investigation (N) is considered to be

innumerable; i.e., $N \gtrsim 20$. Therefore, in the present paper erosion is not a problem even if the overlap factor (b) is somewhat exaggerated [8].) It follows from Eqs. (4)-(6) that

$$[N\eta(\xi,\omega)] \gtrsim [\partial\{k_o(\xi)/m(\xi)\}^{1/2}/(\omega\partial\xi)] \ . \tag{6b}$$

Thus, in general, $\eta(\xi,\omega)$ cannot be set a priori equal to zero. Unless Eq. (5) holds, the transition from the discrete-to-the continuum, which is central to References 1–4, becomes invalid, notwithstanding that in the limit "of a large number of discrete secondary oscillators" the admissible loss factors may be quite small since (N), in Eq. (6), may be quite large [1,8]. (Note that a secondary oscillator = a satellite oscillator.) Equation (6), in terms of Eqs. (13) and (14) of Reference 1, may be stated in the form

$$[N\eta(\xi,\omega)] \gtrsim (\omega_o/\omega)\ (1-\xi)^{-2}\ ; \qquad 0<\xi<1\ , \tag{7}$$

where, again, (N) is the number of satellite oscillators and (ω_o) is defined in Eq. (1). Equations (6) and (7) emphasize that the smallness of the loss factors $\eta(\xi,\omega)$ cannot be arbitrarily assigned; in particular, merely assigning $\eta(\xi,\omega)$ to be zero is not optional.

II. Independence of the contribution to the damping of a master oscillator from the loss factors of coupled satellite oscillators

Employing Eq. (4a), Eq. (2), with the use of Eq. (3), may be cast in the form

$$i\omega M\Big\{[1-(\omega_o/\omega)^2\ (1+i\eta_o)]\ +\ \int_o^1 d\xi\,\overline{m}(\xi)\ \{1+i\eta(\xi,\omega)\}$$

$$\bullet\ [\{1+i\eta(\xi,\omega)\}\ -\ \{\omega/\omega(\xi)\}^2]^{-1}\Big\}\ V_o(\omega)\ =\ P_e(\omega)\ , \tag{8}$$

where $\overline{m}(\xi)\ =\ [m(\xi)/M]$ [1]. Multiplying both sides of Eq. (8) by $V_o^*(\omega)$, the complex conjugate of $V_o(\omega)$, and taking the real part of the resulting equation one obtains

$$\Pi_e(\omega)\ =\ [\omega E_{ok}(\omega)]\ [2(\omega_o/\omega)^2][\eta_o+\eta_s(\omega)]\quad , \tag{9a}$$

where $E_{ok}(\omega)$ is the stored kinetic energy in the master oscillator in situ

$$E_{ok}(\omega)\ =\ (1/2)M\,|V_o(\omega)|^2\ , \tag{10a}$$

$\Pi_e(\omega)$ is the external input power into the master oscillator in situ

$$\Pi_e(\omega)\ =\ \mathrm{Re}\{P_e(\omega)V_o^*(\omega)\}\ , \tag{11}$$

and, finally, $\eta_s(\omega)$ is a measure of the contribution to the damping of the master oscillator by the satellite oscillators

$$\eta_s(\omega)\ =\ (\omega_o/\omega)^2\ \int_o^1 d\xi\overline{m}(\xi)\ [\{\omega(\xi)/\omega\}^2\eta(\xi,\omega)]$$

$$\bullet\ \Big\{[\{\omega(\xi)/\omega\}^2-1]^2\ +\ [\{\omega(\xi)/\omega\}^2\eta(\xi,\omega)]^2\Big\}^{-1}\ . \tag{12a}$$

It is emphasized that $\eta_s(\omega)$ is defined in the mold of the loss factor (η_o) of the master oscillator in isolation. In that sense $\eta_s(\omega)$ is a measure of the contribution to the loss factor of the master oscillator by the presence of the satellite oscillators in the mechanical system in which they are incorporated. [cf. Fig. 1.] A real external input power into a mechanical system requires real damping to exist in that mechanical system; the higher is this damping the higher is the potential injection of power by an external drive. Since only the master oscillator is externally driven, $\Pi_e(\omega)$ is identical to the external power into the mechanical system. Equation (9a) indicates that $\Pi_e(\omega)$ comprises of two parts

$$\Pi_e(\omega) = \Pi_{eo}(\omega) + \Pi_{es}(\omega) \ ; \quad \Pi_{eo}(\omega) = [\omega E_{ok}(\omega)] \ [2(\omega_o / \omega)^2]\eta_o \ ;$$

$$\Pi_{es}(\omega) = [\omega E_{ok}(\omega)] \ [2(\omega_o / \omega)^2] \, \eta_s(\omega) \ , \tag{9b}$$

where $\Pi_{eo}(\omega)$ is the external input power into the mechanical system due to the inherent damping in the master oscillator and $\Pi_{es}(\omega)$ due to the additional damping provided by the satellite oscillators in situ. It transpires, therefore, that the ratio $\mathcal{E}_s(\omega)$ of the external input power that is imparted to the master oscillator due to the presence of the satellite oscillators; i.e., the component $\Pi_{es}(\omega)$, and the total external input power $\Pi_e(\omega)$, is given by

$$\mathcal{E}_s(\omega) \ = \ [\Pi_{es}(\omega) / \Pi_e(\omega)] \ = \ \eta_s(\omega)[\eta_o + \eta_s(\omega)]^{-1} \ = \ [\{\eta_o / \eta_s(\omega)\} + 1]^{-1} \ . \tag{9c}$$

The external input power ratio $\mathcal{E}_s(\omega)$ is depicted, as a function of $[\eta_o / \eta_s(\omega)]$, in Fig. 2. To determine the contribution to the real damping of the mechanical system by the satellite oscillators and hence determine $\mathcal{E}_s(\omega)$, $\eta_s(\omega)$ needs to be evaluated employing Eq. (12a). To

facilitate this evaluation it is useful to render unto this equation a transformation of a variable in the manner

$$g(\xi)d\{\omega(\xi)/\omega\} \;=\; d\xi \;; \qquad g(\xi) \;=\; [\{\partial\omega(\xi)/\omega\}/\partial\xi]^{-1} \;. \tag{13}$$

Applying this tranformation of a variable to Eq. (6a) and Eq. (12a) one obtains

$$[N\eta(x,\omega)] \;\gtrsim\; [g(x)]^{-1} \;; \qquad x(\xi) \;=\; \{\omega(\xi)/\omega\} \;, \tag{6c}$$

$$\eta_s(\omega)=(\omega/\omega_o)^2 \int_o^\infty dx\{[\overline{m}(x)g(x)][x^2\eta(x,\omega)]$$

$$\bullet \; \{[x^2-1]^2+[x^2\eta(x,\omega)]^2\}^{-1} \;, \tag{12b}$$

respectively, and it is recognized, once again, that the validity of Eq. (12b) is predicated on "the discrete-to-the continuum" condition stated in Eq. (6c). Provided the factor $[\overline{m}(x)g(x)]$ in the integrand is well behaved function of (x) and (ω), the integral in Eq. (12b) may be readily executed to yield the result.

$$\eta_s(\omega) \;=\; (\pi/2)(\omega/\omega_o)^2 \; [\overline{m}(\xi_o)g(\xi_o)] \;; \; x(\xi_o) \;=\; \{\omega(\xi_o)/\omega\} \;=\; 1 \;. \tag{12c}$$

Of course, as just discussed, the dimensionless ratio $\eta_s(\omega)$ is a victim of its definition. It is the desire to put $\eta_s(\omega)$ on the same footing with (η_o) that prompted its definition in terms of the stored kinetic energy $E_{ok}(\omega)$ in the master oscillator in situ. In the quantification and division of the external input power $\Pi_e(\omega)$, this definition makes sense. [cf. Eq. (9).] As to whether $\eta_s(\omega)$ qualifies as a loss factor remains to be resolved. Equation (12c) clearly states that $\eta_s(\omega)$ is independent of $\eta(\xi,\omega)$ [8]. Since $\eta_s(\omega)$ is independent of the individual loss factors

of the satellite oscillators, $\mathcal{E}_s(\omega)$, too is independent of these individual loss factors. However, Eq. (6) states that this independence cannot be interpreted to mean that $\eta(\xi,\omega)$ may be set a priori equal to zero; indeed, to validate this independence, $[N\eta(\xi,\omega)]$ must remain finite.

At this stage it may be merely remarked that in Reference 1, the loss factor (η_o) of the master oscillator in isolation is set a priori equal to zero and, therefore, the external input power ratio $\mathcal{E}_s(\omega)$ is set a priori equal to unity. In this case the external input power $\Pi_e(\omega)$ is entirely due to the presence of the satellite oscillators; they alone provide a real damping to the externally driven mechanical system and they alone enable the mechanical system to receive input power from this external drive.

Again, there may be interest in casting Eqs. (6c) and (12c) in terms of Eqs. (13) and (14) of Reference 1. It is first observed that in terms of these referenced equations and the second of Eq. (12c), one derives

$$\xi_o = [1+(\omega_o/\omega)]^{-1} \; ; \qquad (1-\xi_o) = (\omega_o/\omega)\,[1+(\omega_o/\omega)]^{-1} \; , \tag{14a}$$

$$[g(\xi_o)]^{-1} = [(\omega_o/\omega)^{1/2}+(\omega/\omega_o)^{1/2}]^2 \; , \tag{14b}$$

$$\overline{m}(\xi_o) = \mu[1+(\omega_o/\omega)]^2[1+(\omega_o/\omega)^2]^{-1} \; . \tag{14c}$$

Substituting Eq. (14) in Eqs. (6c) and (12c) one obtains

$$[N\eta(\xi_o,\omega) \gtrsim [(\omega/\omega_o)^{1/2} + (\omega/\omega_o)^{1/2}]^2 \; . \tag{6d}$$

$$\eta_s(\omega) = (\mu\pi/2)(\omega/\omega_o)\,[1+(\omega/\omega_o)^2]^{-1} \; ; \quad \{\omega(\xi_o)/\omega\} = 1 \; , \tag{12d}$$

respectively, and in particular setting (ω / ω_o) equal to unity, one obtains

$$[N\eta(\xi_o,\omega_o) \gtrsim 4 \; ; \qquad \eta(\xi_o,\omega_o) \gtrsim (4/N) \; , \tag{6e}$$

$$\eta_s(\omega_o) \; = \; (\mu\pi/4) \; . \tag{12e}$$

In this particular case, the external input power ratio $\mathcal{E}_s(\omega)$ becomes

$$\mathcal{E}_s(\omega_o) \; = \; [(4\eta_o/\pi\mu) \; + \; 1]^{-1} \; , \tag{9d}$$

indicating immediately that a massless satellite oscillators, for which $\mu \; = \; 0$, do not contribute to the external input power into the mechanical system, unless the loss factor (η_o) of the master oscillator, in isolation, is set a priori equal to zero!(?). A question may then arise: which of the two zeros is the more zero! ($[\sin(y)/x]$ as $y \; \to \; 0$ and $x \; \to \; 0$; $y \; = \; (x)^{1/2}$, (x) and $(x)^2$?) The consequence of imposing extreme limits a priori is, thereby, illustrated, in this particular case, as an example.

III. Stored energies in the master oscillators in situ and in the satellite oscillators in situ

The stored kinetic energy $E_{ok}(\omega)$ in the master oscillator in situ is stated in Eq. (10a). In the same vein, the energy $E_o(\omega)$ stored in the master oscillator in situ may be expressed in the form

$$E_o(\omega) \;=\; E_{ok}(\omega)[1+(\omega_o/\omega)^2] \;. \tag{10b}$$

Further yet, the energy $E(\omega)$ stored in the mechanical system, comprising the master oscillator and the coupled satellite oscillators, may be expressed in the form

$$E(\omega) \;=\; E_o(\omega) + E_s(\omega) \;; \qquad E_s(\omega) \;=\; E_o(\omega)\Im_s(\omega) \;, \tag{10c}$$

where $E_s(\omega)$ is the stored energy in the satellite oscillators in situ and

$$[E_s(\omega)/E_o(\omega)] \;=\; \Im_s(\omega) \;=\; [1+(1/2)\eta^2(\omega)]\;[\eta_s'(\omega)/\eta(\omega)] \;;$$

$$\eta(\xi_o,\omega) \;=\; \eta(\omega) \;; \qquad \{\omega(\xi_o)/\omega) \;=\; 1 \;, \tag{15a}$$

$$\eta_s'(\omega) \;=\; 2(\omega_o/\omega)^2\;[1+(\omega_o/\omega)^2]^{-1}\;[\eta_s(\omega)] \;. \tag{15b}$$

[Equation (15) is derived by stating the partial stored energy $E_s(\xi,\omega)$ in the satellite oscillators and performing the integral involved in a manner analogeous to that performed with respect to Eq. (12) [1].] The parameters $\eta_s(\omega)$ and $\eta(\xi,\omega)$, in Eq. (15), are defined by Eq. (12) and the inequality in Eq. (6), respectively. It is recalled that $\eta_s(\omega)$, and, therefore, also $\eta_s'(\omega)$, are largely independent of $\eta(\xi,\omega)$. Thus, provided $\eta(\omega)$ $[=\eta(\xi_o,\omega)]$ is small compared with unity; $\eta^2(\omega) \ll (1/2)$, the ratio $\Im_s(\omega)$, of the stored energies in the satellite oscillators in situ and in the master oscillator, is inversely proportional to $[\eta(\omega)/\eta_s'(\omega)]$. This situation is

depicted in Fig. 3. Clearly, as the satellite oscillators become more and more lossless, i.e., as $[\eta(\omega)/\eta'_s(\omega)]$ approaches zero, the stored energy ratio $\Im_s(\omega)$ increases toward infinity. (This situation, in the appropriate limits, tends to conform with the specific singular finding reported in Reference 1.) Thus, it emerges that whereas the ratio of the external input power $\mathcal{E}_s(\omega)$, defined in Eq. (9c), remains unchanged with changes in the individual loss factor $\eta(\xi_o, \omega)$, the corresponding stored energy ratio $\Im_s(\omega)$ changes inversely to changes in this individual loss factor. What is the significance of this dependence and how is it to be interpreted within the context of dissipation in the mechanical system depicted in Fig. 1?

IV. Dissipated power in the master oscillator and in the satellite oscillators in situ

The external input power component $\Pi_{eo}(\omega)$, defined in Eq. (9b), is identified with the dissipated power in the master oscillator. In terms of the stored energy $E_o(\omega)$ in the master oscillator, the power $\Pi_{eo}(\omega)$ dissipated in that master oscillator may be cast in the form

$$\Pi_{eo}(\omega) = \omega\, \eta_o'(\omega) E_o(\omega); \quad \eta_o'(\omega) = (2\eta_o)(\omega_o/\omega)^2 \; [1+(\omega_o/\omega)^2]^{-1} \; . \tag{16a}$$

Similarly, the external input power component $\Pi_{es}(\omega)$, defined in Eq. (9b), is identified with the dissipated power in the satellite oscillators. In terms of the stored energy $E_s(\omega)$ in the satellite oscillators, the power $\Pi_{es}(\omega)$ dissipated in these oscillators may be cast in the form

$$\Pi_{es}(\omega) \; = \; \omega\eta(\omega) E_s(\omega) \; ; \qquad \eta(\xi_o, \omega) \; = \; \eta(\omega) \; . \tag{16b}$$

and, of course, by definition

$$\Pi_{eo}(\omega) \; + \; \Pi_{es}(\omega) \; = \; \Pi_e(\omega) \; . \tag{16c}$$

Assuming once again that $\eta(\omega)$ is small compared with unity; $\eta^2(\omega) \; << \; (1/2)$, one may establish, from Eqs. (9c), (15) and (16) the following relationships:

$$\mathcal{E}_s(\omega) = [\eta(\omega)\Im_s(\omega)] \; [\eta_o'(\omega) + \eta(\omega)\Im_s(\omega)]^{-1} = \eta_s'(\omega)[\eta_o'(\omega) + \eta_s'(\omega)]^{-1} \; , \tag{17a}$$

$$\Im_s(\omega) = [\eta_o'(\omega)/\eta(\omega)]\, \mathcal{E}_s(\omega)[1 - \mathcal{E}_s(\omega)]^{-1} = [\eta_s'(\omega)/\eta(\omega)] \; . \tag{17b}$$

As previously stated, $\eta_s(\omega)$ and also $\eta_s'(\omega)$, are independent of $\eta(\omega)$. It follows that the dissipated power ratio $\mathcal{E}_s(\omega)$ is also independent of $\eta(\omega)$; the ratio $\mathcal{E}_s(\omega)$ between the power $\Pi_{es}(\omega)$ dissipated in the satellite oscillators in situ and the power $\Pi_e(\omega)$ dissipated in the

mechanical system is independent of the individual loss factors of the satellite oscillators in isolation. [cf. Fig. 2.] One is reminded that $\Pi_e(\omega)$ is also the external input power into the mechanical system as a whole. One the other hand, the stored energy ratio $\Im_s(\omega)$ between the stored energy $E_s(\omega)$, in the satellite oscillators in situ, and the stored energy $E_o(\omega)$, in the master oscillator in situ, is inversely proportional to $\eta(\omega)$; indeed, $\Im_s(\omega)$ increases as $[\eta(\omega)/\eta_s'(\omega)]$ decreases. [cf. Fig. 3.] As Eq. (17) indicates this relationship between $\Im_s(\omega)$ and $[\eta(\omega)/\eta_s'(\omega)]$ maintains unchanged the dissipated power ratio $\mathcal{E}_s(\omega)$. Thus, $\mathcal{E}_s(\omega)$ remains unchanged with respect to changes in $[\eta(\omega)/\eta_s'(\omega)]$, but may change, with changes in $[\eta_o/\eta_s(\omega)] = [\eta_o'(\omega)/\eta_s'(\omega)]$. [cf. Fig. 2.] Indeed, to maintain $\mathcal{E}_s(\omega)$ unchanged with respect to changes in $[\eta(\omega)/\eta_s'(\omega)]$, these changes must be inversely compensated by changes in the stored energy ratio $\Im_s(\omega)$. Moreover, the *conservation of energy* is built into the analysis here presented. The question as to "where did the energy go" is moot in this analysis. The dilemma that has beset the interpretation presented in References 1–4 is, thereby, resolved and is rendered consistent with intuition; intuition wins again! The singularity, that beset the stored energy ratio $\Im_s(\omega)$ in $[\eta(\omega)/\eta_s'(\omega)]$, has caused confusion. The singularity in $\Im_s(\omega)$ exists in the analysis when it is imposed a priori that $[\eta(\omega)/\eta_s'(\omega)]$ is identically equal to zero. In the analysis presented in this paper the asymptotic behaviors of the mechanical system may be investigated when the appropriate limits are approached. This is in contrast to the analyses presented in References 1-4 in which a number of key limits were a priori imposed. Among these key limits is the implication that $[N\,\eta(\omega)]$ may assume the value of zero. This implication defies the condition of modal overlap. [cf. Eq. (6).]

To briefly recapitulate: if the master oscillator is lossless, so that $\eta_o \rightarrow 0$, $\mathcal{E}_s(\omega) \rightarrow 1$ indicating that the dissipation is exclusively occurring in the satellite oscillators. Moreover, this exclusiveness is independent of the individual loss factors $\eta(\omega)$ of these oscillators. Thus, even if $\eta(\omega)$ approaches zero; $\eta(\omega) \rightarrow 0$, the dissipation in the mechanical system is exclusively confined to the satellite oscillators in situ. This situation is maintained because the stored energy ratio $\Im_s(\omega)$ increases inversely to $\eta(\omega)$ so that as $\eta(\omega)$ approaches

zero, the stored energy $E(\omega)$ in the mechanical system resides exclusively in the satellite oscillators; the stored energy $E_o(\omega)$ in the master oscillator pails in comparison with the stored energy $E_s(\omega)$ in the satellite oscillators. The increase in $\Im_s(\omega)$ just compensates for the decrease in $\eta(\omega)$ so that $\mathcal{E}_s(\omega)$ can be maintained at unity. Although the limits are approached smoothly in terms of $\mathcal{E}_s(\omega)$ and $\Im_s(\omega)$, the singularity in $\Im_s(\omega)$, as $\eta(\omega) \rightarrow 0$, may cause analytical discomfort; e.g., is $\eta(\omega) \rightarrow 0$ beyond the value of η_o?; i.e., what are the values of $[\eta_o / \eta(\omega)]$ when $\eta(\omega) \rightarrow 0$? In answering questions of this ilk one is reminded that overriding these questions is the imposition of the condition of modal overlap in which $[N\,\eta(\omega)]$ must remain finite. Without this imposition of modal overlap the analysis cannot proceed from the discrete-to-the continuum which is a basic tenet of the analysis.

A Note: It may come to pass that the imposition of modal overlap is, if (N) is taken large enough, imposed naturally in computer experimental data. This process may be commensurate with that of trying to construct a mechanical system, of the vintage depicted in Fig. 1, with oscillators that are well nigh lossless. Moreover, several papers have attempted to cast the analysis in the temporal (t) domain rather than in the frequency (ω) domain [1,4]. Of course, the analysis in these two different domains can be related by a Fourier transformation. In terms of today's computers, this transformation is easily implemented. However, the author, being computer illiterate, leaves the implementation of the transformation of the analysis, presented herein, into the temporal (t) domain as an exercise to those who are computer literate. The implementation is not an idle exercise; the author is fully aware of the complemental descriptions of the behavior of the mechanical system in the two domains. This complementarity is highly beneficial to the interpretation of this behavior.

V. Loss factors and other dimensionless ratios of dynamic quantities

"All loss factors are useful noise control parameters, but not all dimensionless ratios of dynamic quantities are loss factors; notwithstanding that some such dimensionless ratios may serve as useful indicators of noise control goals and achievements."

It has been commonly assumed that a loss factor $\eta(\omega)$ may be defined in terms of the ratio of the external input power $\Pi_e(\omega)$ and the product of the frequency (ω) and the stored energy $E(\omega)$ that this input power generates; namely

$$\eta(\omega) = \Pi_e(\omega) \{\omega E(\omega)\}^{-1} . \tag{18}$$

Analogeous to Eq. (18), one may employ Eqs. (9) and (10a) to define the dimensionless ratio $\eta_{ok}(\omega)$ in the form

$$\eta_{ok}(\omega) = [\Pi_e(\omega)/\{\omega E_{ok}(\omega)\}] = 2(\omega_o/\omega)^2 [\eta_o + \eta_s(\omega)] . \tag{19a}$$

In the same vein, Eqs. (9a), (10b) and (10c) may be employed to define two more dimensionless ratios; namely

$$\eta_t(\omega) = [\Pi_e(\omega)/\{\omega E_o(\omega)\}] = \eta_{ok}(\omega) [1+(\omega_o/\omega)^2]^{-1} , \tag{19b}$$

$$\eta_e(\omega) = [\Pi_e(\omega)/\{\omega E(\omega)\}] = \eta_t(\omega) [1+\Im_s(\omega)]^{-1} , \tag{19c}$$

where $\Im_s(\omega)$ is stated in Eq. (15) [9,10]. It is established that $\eta_t(\omega)$ may be cast in the form [11]

$$\eta_t(\omega) = [\eta_o' (\omega) + \{\eta(\omega)\Im_s(\omega)\}] , \tag{20a}$$

where $\eta'_o(\omega)$ and $\eta(\omega)$ are stated in Eqs. (16a) and (16b), respectively [11]. Therefore, from Eqs. (19c) and (20a), one obtains

$$\eta_e(\omega) = [\eta'_o(\omega) + \{\eta(\omega)\Im_s(\omega)\}] [1+\Im_s(\omega)]^{-1} \ . \tag{20b}$$

In Eq. (20), $\eta'_o(\omega)$ at resonance, where $(\omega_o / \omega) = 1$, is identified with the loss factor (η_o) of the master oscillator in isolation and $\eta(\omega)$ is identified with the loss factor of the satellite oscillators in isolation and in the distributed placement defined by $\xi = \xi_o$. [cf. Eq. (12d) for the definition of ξ_o.] The parameter $\eta_{ok}(\omega)$, in principle, relegates the measure of damping introduced by the satellite oscillators to the loss factor (η_o) of the master oscillator; i.e., the parameter $\eta_s(\omega)$ equivalently relates, on behalf of the satellite oscillators in situ, to the external input power into the master oscillator. Although $\eta_s(\omega)$ is a useful parameter, its surrogate nature does not quality it to be a loss factor, rendering $\eta_{ok}(\omega)$ also disqualified for this designation. Similarly, the parameter $\eta_t(\omega)$, in principle, relegates the measure of the damping introduced by the satellite oscillators to the loss factor $\eta'_o(\omega)$ of the master oscillator; i.e., the parameter $\{\eta(\omega)\Im_s(\omega)\}$ equivalently relates, on behalf of the satellite oscillators in situ, to the external input power into the master oscillators. (The loss factor $\eta'_o(\omega)$ is equal at resonance, where $(\omega_o / \omega) = 1$, to the loss factor (η_o), both are in reference to the master oscillator in isolation.) The parameter $\eta_t(\omega)$ harbors, again, a surrogate term; i.e., $\{\eta(\omega)\Im_s(\omega)\}$, and, therefore, it, too does not qualify to be a loss factor. Finally, $[\omega\eta_e(\omega)]$ describes the ratio of the external input power $\Pi_e(\omega)$ into the mechanical system to the entire stored energy $E(\omega)$ that this input power generates in the mechanical system. As such $\eta_e(\omega)$ qualifies to be a bonafide loss factor.

In a final note one may attempt to address the following problem:

A mechanical system, comprising a master oscillator, is externally driven. The relationship between the external input power $\Pi_e^o(\omega)$ and the stored energy $E_o^o(\omega)$ generated in the master oscillator may be expressed in the form

$$\Pi_e^o(\omega) = \omega\eta_o'(\omega)E_o^o(\omega) \; ; \qquad \Pi_e^o(\omega) = G^o(\omega)S_f(\omega)\Delta(\omega) \; , \tag{21a}$$

where $G^o(\omega)$ is the conductance of the master oscillator in isolation, $S_f(\omega)$ is the spectral density of the external drive, and $\Delta(\omega)$ is a suitable frequency band [7]. The superscript (o) designates quantities and parameters that pertain to the uncoupled master oscillator. On the other hand, if the mechanical system is extended to include the satellite oscillators, the relationship between the external input power $\Pi_e(\omega)$ and the stored energy $E_o(\omega)$ generated in the master oscillator is expressed in Eq. (19b); namely

$$\Pi_e(\omega) = \omega\eta_t(\omega)E_o(\omega) \; ; \qquad \Pi_e(\omega) = G(\omega)S_f(\omega)\Delta(\omega) \; , \tag{21b}$$

where $G(\omega)$ is the conductance of the extended mechanical system (master oscillator + satellite oscillators) and $S_f(\omega)$ and $\Delta(\omega)$ are identical to those stated in Eq. (21a) [7]. A noise control goal may address minimizing the response of the master oscillator by the extension just proposed. This minimization may be expressed in terms of a reduction in the ratio $[E_o(\omega)/E_o^o(\omega)]$. From Eqs. (21a) and (21b) one obtains

$$[E_o(\omega)/E_o^o(\omega)] = [\Pi_e(\omega)/\Pi_e^o(\omega)]\,[\eta_o'(\omega)/\eta_t(\omega)] \; , \tag{22a}$$

where

$$[\Pi_e(\omega)/\Pi_e^o(\omega)] = G(\omega)/G_o(\omega) \; . \tag{23a}$$

Employing Eqs. (17b) and (20a), Eq. (22a) may be cast in the form

$$[E_o(\omega) / E_o^o(\omega)] \; = \; [\Pi_e(\omega) / \Pi_e^o(\omega)] \; (\eta_o) \; [\eta_o + \eta_s(\omega)]^{-1} \; . \tag{22b}$$

Often one tends to argue that the external input power ratio $[\Pi_e(\omega) / \Pi_e^o(\omega)]$ is largely equal to unity [11,12]. Under this argument, Eq. (22b) reduces to read

$$[E_o(\omega) / E_o^o(\omega)]_1 \; = \; [\eta_o'(\omega) / \eta_t(\omega)] \; \approx \; (\eta_o) \; [\eta_o + \eta_s(\omega)]^{-1} \; , \tag{24a}$$

where $\eta_o'(\omega)$ and $\eta_t(\omega)$ are stated in Eqs. (16a) and (19b), respectively. Then, if $\eta_o << \eta_s(\omega)$, the reduction in the vibrational stored energy in the master oscillator is significant. [cf. Eq. 2.] This reduction may be quoted as the benefit that may be accrued by coupling, to the master oscillator, a set of satellite oscillators; especially if that set yields a value for $[\eta_o / \eta_s(\omega)]$ that is small compared with unity. Then the reduction attained is given by

$$[E_o(\omega) / E_o^o(\omega)]_1 \; \simeq [\eta_o / \eta_s(\omega)] \; << \; 1 \; . \tag{24b}$$

Claims of significant reductions, commensurate with those stated in Eq. (24), have been made [2,3]. However, since $\eta_t(\omega)$ is not a true loss factor, the result stated in Eq. (24) may be suspect; only a true loss factor definitively describes a property of the mechanical system. Indeed, an examination of the external input power ratio $[\Pi_e(\omega) / \Pi_e^o(\omega)]$ reveals that a value of unity for this ratio may not be a viable approximation. Indeed, it was discussed, with respect to Eq. (12c), that the presence of the satellite oscillators may contribute to an increase in the external input power; the increase is associated with a real increase in the damping of the externally driven master oscillator. The measure of the increase is from (η_o) in isolation to $[\eta_o + \eta_s(\omega)]$ when the master oscillator is coupled to the set of satellite oscillators.

[cf. Eq. (9a).] A statistical energy analysis (SEA) estimate, via Eq. (23a), the external input power ratio to be

$$[G(\omega)/G^o(\omega)] \approx [1+\Im_s(\omega)]\ [1+\{(M_s/M)\xi_o^s(\omega)\}]^{-1}\ ; \qquad \xi_o^s(\omega)<1\ , \qquad (23b)$$

where M_s is the total mass in the satellite oscillators and $\xi_o^s(\omega)$ is the modal coupling strength between the satellite oscillators and the host master oscillator [12]. (In this context, $\Im_s(\omega)$ is the corresponding global coupling strength [12].) Substituting Eq. (23b) in Eq. (22a) yields

$$[E_o(\omega)/E_o^o(\omega)] \ \approx\ [\eta_o'(\omega)/\eta_e(\omega)]\ [1+(M_s/M)\xi_o^s(\omega)\}]^{-1}\ , \qquad (22c)$$

where $\eta_o'(\omega)$ and $\eta_e(\omega)$ are stated in Eqs. (16a) and (19c), respectively. In this connection, except for a few extreme cases, designing a massive set of satellite oscillators, for which $(M_s/M)\ \gtrsim (1/3)$, in order to beneficially decrease the second factor on the right of Eq. (22c), is considered a bad noise control proposal. (A tale of a tail that wags the dog!) Equation (22c) may then be approximated to read

$$[E_o(\omega)/E_o^o(\omega)]\ \simeq [E_o(\omega)/E_o^o(\omega)]_1\ [1+\Im_s(\omega)]$$

$$\simeq \{\eta_o/\eta_s(\omega)\}\ [1+\{\eta_o/\eta_s(\omega)\}]^{-1}\ [1+\{\eta_s'(\omega)/\eta(\omega)\}]\ ;\ (M_s/M)\lesssim(1/3)\ , \qquad (22d)$$

where use is made of Eqs. (15b), (17b) and (24). To ensure that the gain in reduction remains largely the same as stated in Eq. (24), $\Im_s(\omega)$ must be designed to be of the order of or less than unity, requiring, therefore, that $[\eta_s'(\omega)/\eta(\omega)]\ \lesssim 1$. According to the preceding arguments, to satisfy such an inequality is a tall order indeed. A less drastic design criterion is to require that

$[\eta_o'(\omega)/\eta(\omega)]$ be small compared with unity, leaving, thereby, $[\eta_s'(\omega)/\eta(\omega)]$ to assume a natural value that is usually far in excess of unity [8]. In this case, Eq. (22d) yields

$$[E_o(\omega)/E_o^o(\omega)] \simeq [\eta_o'(\omega)/\eta(\omega)] \; ; \qquad [\eta_o'(\omega)/\eta(\omega)] \ll 1 \; ;$$

$$[\eta_s'(\omega)/\eta(\omega)] \gg 1 \; . \tag{22e}$$

Clearly, in this design the reduction implied by $[E_o(\omega)/E_o^o(\omega)]_1$; as stated in Eq. (24b), is an over estimate of the reduction that is attained by the actual stored energy ratio $[E_o(\omega)/E_o^o(\omega)]$; as stated in Eq. (22e). One recognizes that this over estimate could be significantly high [11]. Nonetheless, to end on a more positive note, it is quite remarkable that the analysis provides such simple and direct design criteria in which only rough estimates of the parameters and ratios of quantities are required. Moreover, assessments of these parameters and ratios are required largely on the basis as to *which is greater than which.*

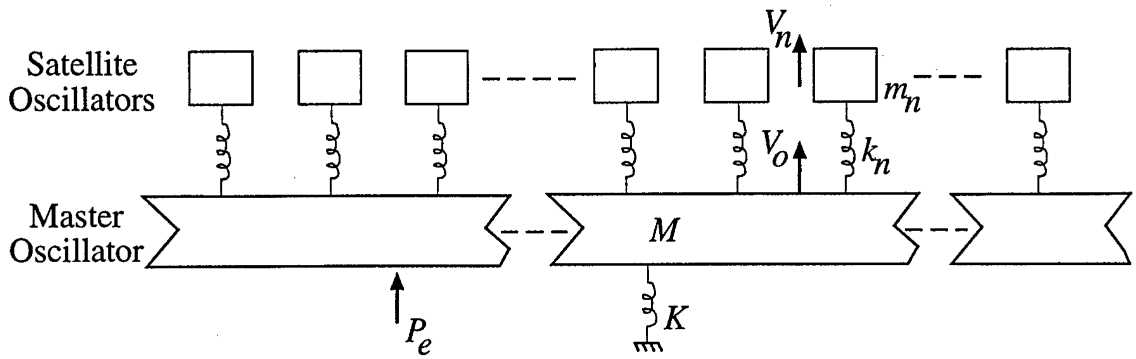

Fig. 1. Master oscillator coupled to a set of distributed satellite oscillators. The master oscillator is defined by the mass (M) and the stiffness (K) and the $(n)th$ satellite oscillator by the mass (m_n) and the stiffness (k_n). Only the master oscillator is driven, by the external drive $P_e(\omega)$, generating the response $V_o(\omega)$ in the master oscillator and the response $V_n(\omega)$ in the $(n)th$ satellite oscillator.

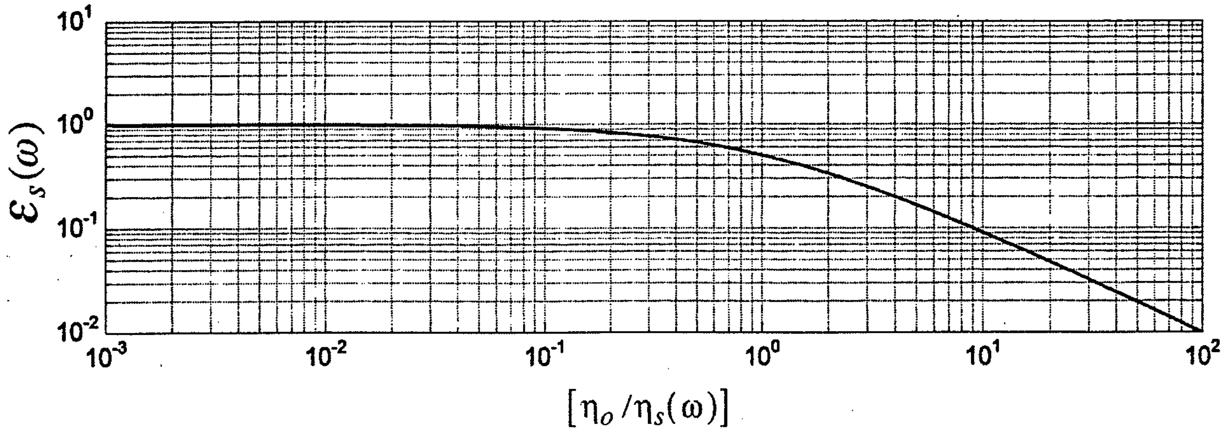

Fig. 2. The power ratio $[\Pi_{es}(\omega) / \Pi_e(\omega)] = \varepsilon_s(\omega)$ dissipated in the satellite oscillators in situ to the external input power $\Pi_e(\omega)$ into the mechanical system, as a function of the ratio $[\eta_o(\omega) / \eta_s(\omega)]$. (η_o) is the loss factor of the master oscillator in isolation and $\eta_s(\omega)$ is the corresponding contribution to the damping of the master oscillator due to the satellite oscillators in situ.

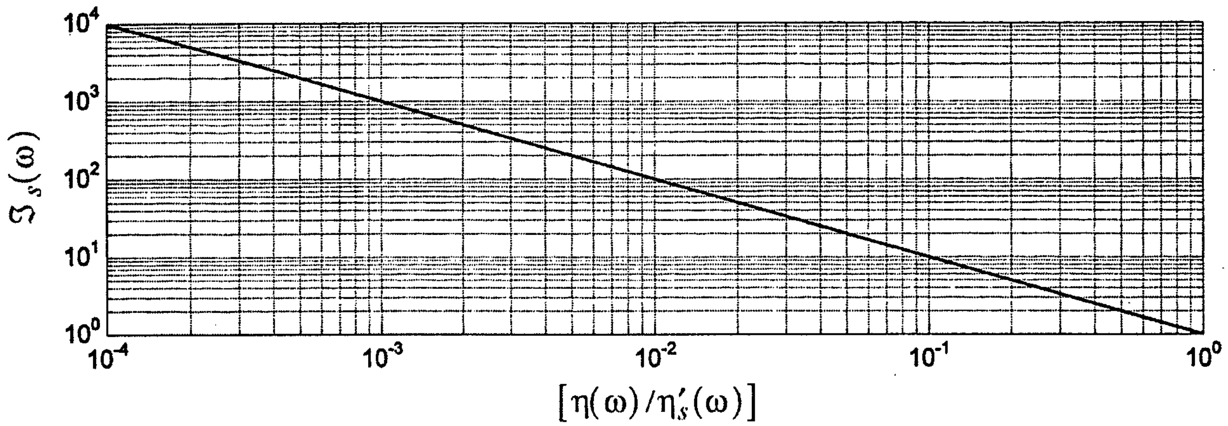

Fig. 3. The ratio $\Im_s(\omega)$ of the stored energy $E_s(\omega)$ residing in the satellite oscillators in situ to the stored energy $E_o(\omega)$ residing in the master oscillators in situ as a function of the $[\eta(\omega)/\eta_s'(\omega)]$.

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